Available online at www.sciencedirect.com
ScienceDirect
Cold Regions Science and Technology 46 (2006) 192 – 203
cold regions science and technology
www.elsevier.com/locate/coldregions

A Bayesian probabilistic framework for avalanche modelling based on observations

Daniel Straub [a,*], Adrienne Grêt-Regamey [b]

[a] *Department of Civil and Environmental Engineering, University of California, Berkeley, CA 94720-1710, USA*
[b] *WSL Swiss Federal Institute for Snow and Avalanche Research, Division, 7260 Davos, Switzerland and ETH Zürich, Institute for Spatial and Landscape Planning, 8093 Zürich, Switzerland*

Received 30 May 2006; accepted 22 August 2006

Abstract

Applied avalanche models are based on parameters which cannot be measured directly. As a consequence, these models are associated with large uncertainties, which must be addressed in risk assessment. To this end, we present an integral probabilistic framework for the modelling of avalanche hazards. The framework is based on a deterministic dynamic avalanche model, which is combined with an explicit representation of the different parameter uncertainties. The probability distribution of these uncertainties is then determined from observations of avalanches in the area under investigation through Bayesian inference. This framework facilitates the consistent combination of physical and empirical avalanche models with the available observations and expert knowledge. The resulting probabilistic spatial model can serve as a basis for hazard maping and spatial risk assessment. In this paper, the new model is applied to a case study in a test area located in the Swiss Alps.

Keywords: Avalanches; Natural hazards; Bayesian analysis; Risk assessment; Uncertainty modelling; Land-use planning

1. Introduction

Mountainous areas are subject to various gravitational hazards, including avalanches, debris flows, land slides and rock-falls. It is increasingly recognized by decision makers that the effective and rational management of such natural hazards requires a risk-based strategy which explicitly addresses the involved uncertainties together with the consequences of such events, e.g. PLANAT (2004). An important part of such a strategy is constructing a probabilistic hazard model, which should be based on all available information, including physical models and observations of past events, following the principles of Bayesian decision theory, for more information refer to DeGroot (1970) or Benjamin and Cornell (1970).

The purpose of this paper is to provide such a probabilistic hazard model for avalanche hazards. The probabilistic model presented in this paper can be considered as a framework to accommodate any existing one- or two-dimensional phenomenological avalanche model. To demonstrate the implementation of this method, we apply a state-of-the-art two-dimensional dynamic simulation model, the AVAL-2D, used in Switzerland for avalanche prediction and hazard zoning (Gruber, 1999). In Grêt-Regamey and Straub (2006), we demonstrate how the model may be included in a risk analysis framework using Bayesian Networks.

* Corresponding author.
E-mail addresses: straub@ce.berkeley.edu (D. Straub), gret@nsl.ethz.ch (A. Grêt-Regamey).

0165-232X/$ - see front matter © 2006 Elsevier B.V. All rights reserved.
doi:10.1016/j.coldregions.2006.08.024

Avalanche modelling is hindered by the difficulties involved in physical avalanche modelling, the strong limitations of empirical relationships and the limited availability of site-specific information. For this reason, probabilistic approaches to avalanche hazard modelling have been proposed in the past. A broad overview on the work in this area is provided in Ancey et al. (2004). In Harbitz et al. (2001), observations (respectively no-observations) are used to estimate the release probability, whereas the avalanche run-out distance is determined using a statistical model based on observations at other sites. In the papers by Barbolini et al. (2002, 2003), only the avalanche release is considered probabilistically, whereas the parameters of the dynamic avalanche model (μ and ξ) are considered deterministically. The most comprehensive probabilistic model is presented in Ancey et al. (2004), which calibrates both the release probability and the parameter μ to observations at the site. We propose to consider the existing models in a Bayesian framework. Due to its flexibility, the Bayesian model enables us to account for all available information in a consistent way (in particular the combination of information from different sources is facilitated). It furthermore allows the joint representation of all uncertainties in the model (i.e. uncertainties in both the release and the flow process). Bayesian inference is also applied in Ancey (2005) in order to investigate the dependency of the friction parameters in the dynamic avalanche model on the avalanche volume, with a focus on investigating generally valid relationships among the model parameters. Considering the large uncertainties involved in avalanche modelling, the existence of such relationships is questioned by many, which is in accordance with the conclusions drawn in Ancey (2005). For this reason, we focus on model updating using site-specific observations. Our goal is to provide a practically applicable yet scientifically sound framework which facilitates rational engineering decision making for avalanche hazards.

The problems and solutions presented here are also relevant to other gravitational natural hazards, such as landslides and rock-falls. To our knowledge, the approach introduced in this paper has so far not been introduced for those hazards, although it has been outlined in Schubert et al. (2005) for rock-falls.

2. Probabilistic modelling of the avalanche run-out distance

The avalanche process can be divided into the release process and the dynamic avalanche flow. In the following, first a probabilistic model for the release process and thereafter for the avalanche flow will be presented. Both parts are associated with uncertainties, which must be explicitly addressed in a fully probabilistic approach. These can be categorized into aleatory uncertainties (related to the inherent randomness of the processes) and epistemic uncertainties (related to our incomplete knowledge of the world) as discussed in Hall (2003), Wen et al. (2003) and Faber (2005). This distinction is relevant because, for a given model, only the epistemic uncertainties can be reduced through observations.

2.1. Reference period

The ultimate goal of the probabilistic avalanche model is consistent assessment and management of avalanche risks in the built environment. Such risks are best expressed as annual risks, see Rackwitz (2000). Therefore, in the following all probabilities have reference period of 1 year, whereby a year is considered to range from summer to summer, to include the entire avalanche season. The use of annual probabilities is not trivial and has some implications. First, we neglect the probability that two damaging avalanches occur within 1 year, and second, we neglect the fact that the largest avalanche does not necessarily coincide with the largest snow depth in the release zone. These simplifications are commonly made in avalanche risk assessments.

2.2. Avalanche release process

In the two-dimensional physical avalanche model, the avalanche release process is represented by the annual maximum detached snow volume, described by the area A_R and the snow depth H of the detached snow mass. This is then used as an input in the dynamic avalanche flow model. Both A_R and H are uncertain and should be modelled by probabilistic variables, as pointed out in Barbolini et al. (2002).

Here, we take basis in the release scenarios identified as part of the standard procedures for hazard mapping in Switzerland, see Gruber (2001) and Maggioni and Gruber (2003). The different release scenarios are determined based on meteorological data, topography, past observations of avalanches and expert opinions and are assumed to be representative for release conditions with annual exceedance probabilities of $1/T_R$; typical values of the return period T_R are 10 years, 30 years, 100 years, 300 years. The different release scenarios, which include the spatial description of the release area and the snow depth, are denoted by r_i (e.g., scenario r_{30} represents the snow release volume which is expected to be exceeded every 30 years).

The annual probability of occurrence of each scenario, $p_R(r)=\mathrm{P}(R=r)$, is a direct function of the corresponding return period. Note that, although this is often done in risk analysis, the occurrence probability must not be mistaken for the exceedance probability: $p_R(r_{100})$ is in general not equal to 1/100. Instead the scenario r_{100} represents the release area sizes with exceedance probabilities around 1/100, its probability, therefore, depends on the next lower and next higher scenario. As an example, assuming that r_{100} is representative for all scenarios with annual exceedance probability between 1/50 and 1/200, its annual probability of occurrence is 1/50–1/200 = 0.015.

In addition to the typically evaluated release scenarios, we introduce the scenario r_1 with an annual exceedance frequency equal to 1. In most cases, it is not required to compute the scenario r_1 explicitly, because avalanches that occur every year will not lead to damages in the built environment. It can then be assumed that r_1 corresponds to no avalanche. The probability that no relevant avalanche occurs in a given year is highly site specific and must be determined based on the available observations as part of the Bayesian inference which will be introduced later.

2.3. Uncertainty on the avalanche release process

In the avalanche release process model, as presented above, it is assumed that in any year the release scenario r occurs with a probability of $p_R(r)$. It is uncertain which release scenario will occur, e.g., next year, and it is not possible to reduce this (aleatory) uncertainty beforehand. However, this is not the only uncertainty, because in addition, we do not know the exact value of $p_R(r)$. It is proposed to explicitly include this epistemic uncertainty on $p_R(r)$ in the analysis, because this uncertainty can be reduced when additional information becomes available. This is done by allowing for different alternative exceedance probability curves for R and by assigning probabilities to all alternatives. Similar concepts may be found in earthquake engineering, e.g., Baker and Cornell (2003), and have also been proposed for rock-fall modelling in Straub (2005). The set of different alternative exceedance probability curves is represented by the random variable Θ_R, which here can take five values θ_{R1} to θ_{R5}, whereby θ_{R3} corresponds to the mean curve.

A problem with this representation of model uncertainty is that the probabilities of occurrence of the different scenarios (large and small avalanches) become correlated. Often, different mechanisms are underlying the scenarios and there is no justification for this dependency which is introduced through the analysis. For the present avalanche model, this is relevant for the nil-scenario r_1, as its probability is dependent on the definition of a relevant avalanche (in our example all scenarios r which are equal or larger than r_{10}). For this reason, it is proposed to represent the probability of r_1 by a separate random variable Φ (which, by definition, also influences the probability of r_{10}).

The epistemic uncertainty on $p_R(r)$ is thus characterized by the two random variables Θ_R and Φ.

The resulting probabilistic model for R is presented in Table 1, which summarizes the annual probabilities for the different scenarios as a function of Θ_R and Φ (whose realizations are denoted by φ).

With this model, the annual probability of any scenario r, conditional on φ and θ_R, is written as $p_{R|\theta_R,\varphi}(r)$ and, following the total probability theorem, the unconditional annual probability is

$$p_R(r) = \sum_{\Phi}\sum_{\Theta_R} p_{R|\theta_R,\varphi}(r)\, p_{\Theta_R,\Phi}(\theta_R,\varphi) \quad (1)$$

$p_{\Theta_R,\Phi}(\theta_R,\varphi)$ is the joint probability of $\{\Theta_R=\theta_R\}$ and $\{\Phi=\varphi\}$. It is pointed out that a-priori it is sufficient to consider the unconditional $p_R(r)$ without explicitly accounting for the (epistemic) uncertainty introduced through Θ_R and Φ. However, these uncertainties are important when accounting for past observations, i.e., in the a-posteriori model, because it is possible to learn on Θ_R and Φ, as demonstrated later.

2.4. Dynamic avalanche model

The applied model, AVAL-2D from the Swiss Federal Institute for Snow and Avalanche Research

Table 1
Probabilities of the different release scenarios for different Θ_R and Φ

Release scenarios	Probability of occurrence of R, conditional on the realizations of the model uncertainties Θ_R and Φ				
	$p_{R\vert\theta_{R1},\varphi}(r)$	$p_{R\vert\theta_{R2},\varphi}(r)$	$p_{r\vert\theta_{R3},\varphi}(r)$	$p_{R\vert\theta_{R4},\varphi}(r)$	$p_{R\vert\theta_{R5},\varphi}(r)$
r_1	$1-\varphi$	$1-\varphi$	$1-\varphi$	$1-\varphi$	$1-\varphi$
r_{10}	$\varphi-0.016$	$\varphi-0.033$	$\varphi-0.05$	$\varphi-0.067$	$\varphi-0.084$
r_{30}	0.01	0.02	0.03	0.04	0.05
r_{100}	0.005	0.01	0.015	0.02	0.025
r_{300}	0.001	0.003	0.005	0.007	0.009

(SLF), is a two-dimensional dynamic avalanche program that identifies the sizes of avalanche release zones, predicts run-out distances, flow velocities and impact pressures of dense snow avalanches. The flow simulation model employs a modified "Voellmy-fluid" flow law, see Bartelt et al. (1999), which assumes small shear strains in the flow body, and concentration of the flow resistances at the base of the avalanche. The flow resistance is determined by a dry-Coulomb type friction (μ) and a velocity squared friction (ξ). The latter is here modelled as a deterministic parameter, with different values for different slope angles, topographical classifications (such as open, confined, gully or flat) and surfaces (e.g., a value of 400 m/s^2 is assumed in forest areas). The parameter μ, which, as noted in Barbolini et al. (2000), has the largest influence on the run-out distance, is modelled probabilistically. Because different μ-values are used in the avalanche model for the different topographical classifications, we do not represent μ by a single random variable. Instead, we identify nine different parameter scenarios $\theta_{\mu_1},\ldots, \theta_{\mu_9}$, based on the original choice of the avalanche modelling expert. Each of these scenarios correspond to a set of values for μ (μ_A for open terrain, μ_B for confined paths, etc.). It is assumed that one of these scenarios contains the "true" parameter values, yet it is unclear which. This epistemic uncertainty is taken into account by letting the parameter scenario be a random variable Θ_μ with realisations $\theta_{\mu 1},\ldots, \theta_{\mu 9}$. In Fig. 1, the influence of Θ_μ on the calculated avalanches is illustrated for a given release scenario.

Conceptually, the dynamic avalanche model is in the following interpreted as a deterministic function f_{Aval}. f_{Aval} gives the annual maximum pressure P for any spatial coordinate $\mathbf{u}$ on a 5 m × 5 m raster. The input parameters to the model are divided into a set of random variables $\mathbf{\Theta} = \{\Theta_R, \Phi, R, \Theta_\mu\}$ and a set of deterministic constants $\mathbf{c}$. The latter include all inputs to the model that are considered as deterministic, such as topography or the parameter ξ. Clearly, it depends on the chosen model which of the parameters belong to $\mathbf{c}$, e.g., ξ may also be modelled as a stochastic variable. Hereafter, we omit the term $\mathbf{c}$, which is considered to be included in the function f_{Aval}. The avalanche model is subject to an error ε, because the model is only a simplified representation of reality and because some of the input parameters to the model considered as deterministic are actually uncertain. The error ε is here assumed to be additive in P and it follows that the annual

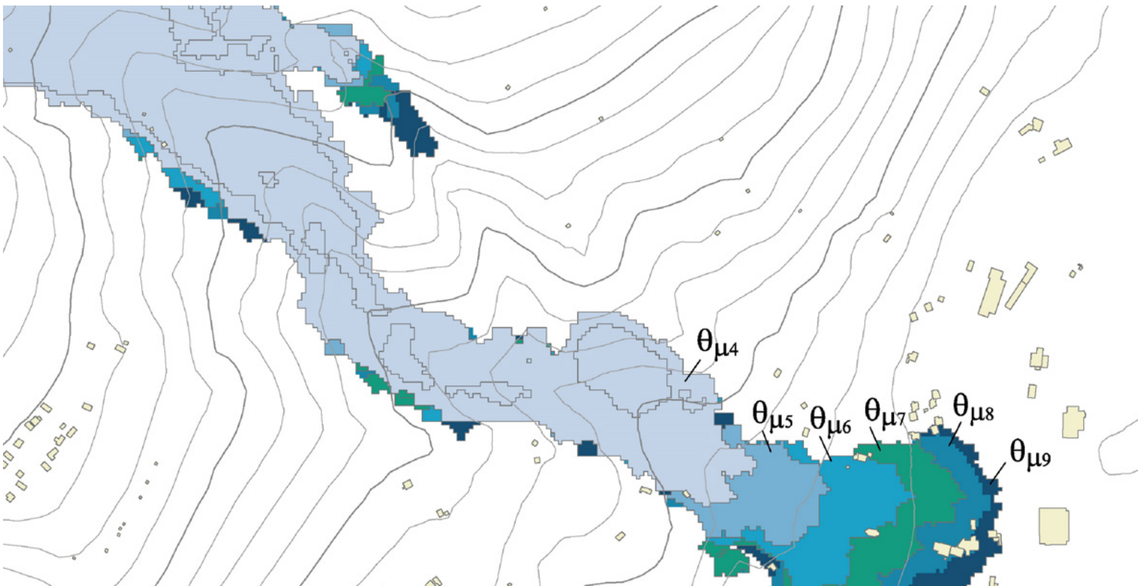

Fig. 1. Example outcome of the AVAL-2D model for different Θ_μ and one release scenario r_{100} (contour lines are shown at 25 m intervals).

maximum pressure at location $\mathbf{u}$ is, for given values of the random variables $\boldsymbol{\Theta}$:

$$P(\mathbf{u},\boldsymbol{\theta}) = f_{\mathrm{AVAL}}(\mathbf{u},\boldsymbol{\theta}) + \varepsilon(\mathbf{u}) \tag{2}$$

Because $\varepsilon=\{\varepsilon(\mathbf{u}):\mathbf{u}\in A\}$ is a random field in the area under investigation A, $P(\boldsymbol{\theta})=\{P(\mathbf{u},\boldsymbol{\theta}):\mathbf{u}\in A\}$ is also a random field. In practice, the determination of the statistics of ε is very difficult, but ε must be included for the purpose of the Bayesian inference, as will be described later. However, for the purpose of the spatially explicit risk assessment presented in Grêt-Regamey and Straub (2006), it is assumed that $\varepsilon=0$ for all $\mathbf{u}$.

2.5. Summary of the probabilistic avalanche model

The probabilistic model as presented above can be summarized in the form of a Bayesian network (BN), Fig. 2. A BN is a directed acyclic graph; the nodes in the graph correspond to the random variables and the arcs represent the dependency structure of the problem, see Pearl (1988) or Jensen (2001).

In Fig. 2, it is assumed that the different epistemic uncertainties in the model $\{\Theta_R, \Phi, \Theta_\mu\}$ are conditionally independent. Releasing this assumption is straightforward; in fact, it will be shown that in the posterior model (i.e., after consideration of the observations) these uncertainties become dependent.

Because we use discrete random variables $\boldsymbol{\Theta}$ to represent the parameter uncertainties, these can be described by the joint probability mass function (pmf) $p_{\boldsymbol{\Theta}}(\boldsymbol{\theta})$. To enhance readability, in the remaining text we will refer to pmf's as probabilities.

3. Bayesian analysis of the model based on records of observed avalanches

Bayes' theorem allows for updating a prior probabilistic model of the variables $\boldsymbol{\Theta}$ with observations $\mathbf{q}$ of the process under consideration, resulting in the posterior probability, $p_{\boldsymbol{\Theta}|\mathbf{q}}(\boldsymbol{\theta})$. Bayes' theorem is, in its general form, given by

$$p_{\boldsymbol{\Theta}|\mathbf{q}}(\theta) \propto L(\boldsymbol{\theta}|\mathbf{q}) p_{\boldsymbol{\Theta}}(\boldsymbol{\theta}) \tag{3}$$

The proportionality constant may be determined from the condition that summation of $p_{\boldsymbol{\Theta}|\mathbf{q}}(\boldsymbol{\theta})$ over the entire domain of $\boldsymbol{\Theta}$ must yield one. $L(\boldsymbol{\theta}|\mathbf{q})$ is the likelihood function which describes how likely the observed realizations of the random process $\mathbf{q}$ are, given a particular value of the variables, $\boldsymbol{\theta}$.

The observations are the dimensions of past avalanches. To be consistent with the model, only the largest avalanche observed in any winter season is considered as an observation. Ideally, the information is available for any of the n years in the records and for the entire area A as $\mathbf{q}(t)=\{\mathbf{q}(\mathbf{u},t):\mathbf{u}\in A\}$, where $\mathbf{q}(\mathbf{u},t)$ takes value 1 if the largest avalanche has been observed at the coordinate $\mathbf{u}$ in year t, and value 0 if the avalanche has not been observed at this location. If it is assumed that the maximum avalanches are independent at different years for given parameter values $\boldsymbol{\theta}$, the likelihood function for the entire set of avalanche records can be written as

$$L(\boldsymbol{\theta}|\mathbf{q}) = \prod_{t=1}^{n} \mathrm{P}(I[P(\mathbf{u},\boldsymbol{\theta})] = \mathbf{q}(\mathbf{u},t) : \mathbf{u}\in A) \tag{4}$$

$I[\cdot]$ is the indicator function, which takes value 1 if its argument is larger than 0 andvalue 0 otherwise. If the error term $\varepsilon(\mathbf{u})$ in Eq. (2) is neglected, the likelihood function, Eq. (4), will in general be zero, meaning that the observation is impossible given the model and its parameters. This is because the avalanche model cannot exactly match the observed avalanche, as some inaccuracies and errors are always present. The random error term must thus be introduced in the model for the Bayesian analysis. However, it is very difficult to define an error random field ε (which is non-Gaussian and is dependent on topography) and to consequently evaluate Eq. (4). For this reason, the observations of avalanches will be considered one-dimensionally along the avalanche path in the following. Note that we do not include the uncertainty related to the observations in Eq. (4). For practical purposes, this uncertainty can here be considered as part of the model uncertainty $\varepsilon(\mathbf{u})$.

For the purpose of model updating, the output of the avalanche model as a function of the input variables $\boldsymbol{\theta}$ can now be reduced to the calculated run-out distance of

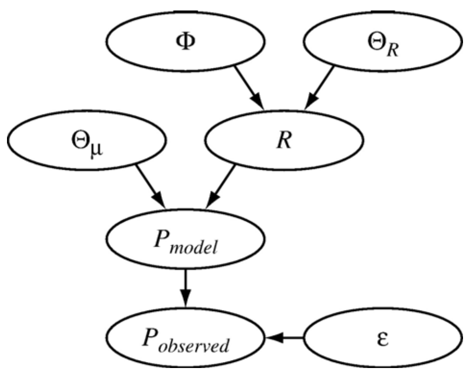

Fig. 2. The a-priori probabilistic avalanche model represented by a Bayesian network.

the avalanche, termed d_{AVAL}. In analogy with Eq. (2), the annual maximum run-out distance D is then defined as

$$D(\boldsymbol{\theta}) = d_{\mathrm{AVAL}}(\boldsymbol{\theta}) + \delta \tag{5}$$

δ is the error term, which is additive in the run-out distance. For simplicity, it is assumed that δ includes both the model error related to d_{AVAL} and the measurement errors pertaining to the observations. From Eq. (5), the probability density function (pdf) of the run-out distance conditional on $\boldsymbol{\theta}$ can be determined as

$$f_{D|\boldsymbol{\theta}}(d) = f_\delta(d - d_{\mathrm{AVAL}}(\boldsymbol{\theta})) \tag{6}$$

with $f_\delta(d)$ being the pdf of the error term δ. The observations $\mathbf{q}(t)$ can now be reduced to $o(t)$, which denotes the observed run-out distance of the largest avalanche in year t along the path. The corresponding likelihood function is

$$L(\boldsymbol{\theta}|\mathbf{q}) = \prod_{t=1}^{n} \mathrm{P}(D(\boldsymbol{\theta}) = o(t)) = \prod_{t=1}^{n} f_{D|\boldsymbol{\theta}}(o(t)) \tag{7}$$

Bayes' theorem, Eq. (3), can now be applied to update the prior model of the variables $\boldsymbol{\Theta}$. The posterior joint probability of the variables is therefore obtained by

$$p_{\boldsymbol{\Theta}|\mathbf{q}}(\boldsymbol{\theta}) \propto \left(\prod_{t=1}^{n} f_\delta(o(t) - d_{\mathrm{AVAL}}(\boldsymbol{\theta})) \right) p_{\boldsymbol{\Theta}}(\boldsymbol{\theta}) \tag{8}$$

For the probabilistic model presented earlier, $\boldsymbol{\Theta}$ consists of the three variables Θ_μ, Θ_R and Φ, the first related to the dynamic avalanche model, the latter two related to the release scenarios. A-priori these three variables are here considered as independent, the prior joint probability is thus obtained by multiplication of the three marginal pmfs:

$$\begin{aligned} p_{\boldsymbol{\Theta}}(\boldsymbol{\theta}) &= p_{\Theta_\mu,\Theta_R,\Phi}(\theta_\mu, \theta_R, \varphi) \\ &= p_{\Theta_\mu}(\theta_\mu) p_{\Theta_R}(\theta_R) p_\Phi(\varphi) \end{aligned} \tag{9}$$

Because of the updating of the prior model with observations, the three variables are dependent in the posterior model. This dependency must be included in the risk analysis. Furthermore, as may be observed from Fig. 2, the three variables Θ_R, Φ and R may be collapsed into the single variable R, see also Jensen (2001). Because R is, for given Θ_R and Φ, independent of the observations $\mathbf{q}$, the joint probabilities of R and Θ_μ are obtained from the application of the total probability theorem as

$$p_{R,\Theta_\mu|\mathbf{q}}(r, \theta_\mu) = \sum_{\Theta_R} \sum_{\Phi} p_{R|\theta_R,\varphi}(r) p_{\Theta_\mu,\Theta_R,\Phi|\mathbf{q}}(\theta_\mu, \theta_R, \varphi) \tag{10}$$

The marginal probability of R is obtained by summation of $p_{R,\Theta_\mu|\mathbf{q}}(r,\theta_\mu)$ over the domain of Θ_μ:

$$p_{R|\mathbf{q}}(r) = \sum_{\Theta_\mu} p_{R,\Theta_\mu|\mathbf{q}}(r, \theta_\mu) \tag{11}$$

The dependency between Θ_μ and R in the posterior model can be represented through the conditional probability of Θ_μ on R, which is calculated by

$$p_{\Theta_\mu|r,\mathbf{q}}(\theta_\mu) = \frac{p_{R,\Theta_\mu|\mathbf{q}}(r, \theta_\mu)}{p_{R|\mathbf{q}}(r)} \tag{12}$$

When neglecting the error term and based on the posterior model of the parameters, the BN to be applied in risk assessment and hazard mapping is shown in Fig. 3.

4. Application

The Bayesian probabilistic avalanche model is applied to the Frauentobel area located in Davos, Switzerland, where one particular avalanche path is studied. For the Bayesian updating of the two-dimensional model by means of the above described one-dimensional approach, using the record of observed maximum avalanches, it is required to determine a representative avalanche flow line. Here, the path is chosen to follow the stream network and is identified using a GIS software.

For the purpose of traceability, the outcomes of the dynamic avalanche model are presented for different values of the input parameters (the model variables) in Table 2.

The observed run-out distances between 1950 and 2003, utilized in the analysis, are based on unpublished data from the SLF Davos. They are collected in the following array: $\mathbf{q} = \{o(t): t \in [1950, \ldots, 2003]\} = \{0$, 635 m, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 630 m, 0, 0, 0, 2210 m, 0, 0, 0, 0, 0, 0, 1105 m, 215 m, 0, 0, 815 m, 1055 m, 1910 m, 0, 0, 2115 m, 1275 m, 1135 m, 1080 m, 680 m, 0, 0, 0, 0, 0, 740 m, 0, 220 m, 0, 0, 1780 m, 1195 m, 0, 0, 0$\}$.

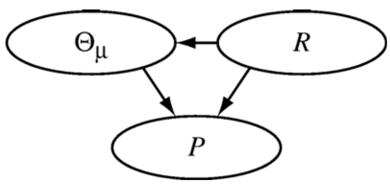

Fig. 3. The Bayesian network representing the posterior avalanche model.

Table 2
Calculated run-out distance d_{AVAL} [m] for different model parameter scenarios and different release scenarios (evaluated with AVAL-2D)

Model parameter scenarious Θ_μ	Release scenarios R				
	r_1	r_{10}	r_{30}	r_{100}	r_{300}
$\theta_{\mu 1}$	0	1370	1425	1455	1870
$\theta_{\mu 2}$	0	1455	1875	1830	1875
$\theta_{\mu 3}$	0	1755	1880	1870	1865
$\theta_{\mu 4}$	0	1815	1885	1970	1895
$\theta_{\mu 5}$	0	1860	2000	2195	2000
$\theta_{\mu 6}$	0	1925	2085	2190	2065
$\theta_{\mu 7}$	0	1935	2135	2185	2025
$\theta_{\mu 8}$	0	2100	2195	2190	2195
$\theta_{\mu 9}$	0	2155	2265	2195	2215

The observations cannot be used directly in the analysis. This is because no release scenarios smaller than r_{10} have been evaluated and it has been assumed that all release scenarios $r<r_{10}$ are represented by r_1, the nil scenario in the model. Yet, in the avalanche records, many of the observed avalanches are associated with release scenarios with a return period lower than 10 years. In order for the observations to correspond to the model, we introduce a threshold o_T: The records which are lower than or equal to the threshold are not considered directly following Eqs. (7) and (8). Instead, only the information that the observation is smaller than the threshold is utilized, i.e., these records are considered as censored data. It follows that, for all observations $o(t)\leq o_T$, the likelihood term (corresponding to Eq. (7)) is

$$L(\boldsymbol{\theta}|o(t)) = \mathrm{P}(D(\boldsymbol{\theta})\leq o_T) = F_{D|\boldsymbol{\theta}}(o_T) \tag{13}$$

with $F_{D|\boldsymbol{\theta}}(d)$ being the cumulative probability distribution of the run-out distance D, conditional on the model parameters $\boldsymbol{\theta}$. Here, o_T has been chosen as 1300 m, as this is lower than the lowest calculated run-out distance for the r_{10} scenario. A sensitivity study has been performed to check the appropriateness of this choice.

For the analysis, it is required to specify prior probabilities for Θ_μ, Θ_R and Φ. This choice, which is far from trivial, is discussed later. For the final results, it has been decided to use uniform probability distributions as prior probabilities for Θ_μ and Φ and the prior probability according to Table 3 for Θ_R. Note that the

Table 3
Prior probability distribution for Θ_R

$p_{\Theta_R}(\theta_{R1})$	$p_{\Theta_R}(\theta_{R2})$	$p_{\Theta_R}(\theta_{R3})$	$p_{\Theta_R}(\theta_{R4})$	$p_{\Theta_R}(\theta_{R5})$
0.05	0.2	0.5	0.2	0.05

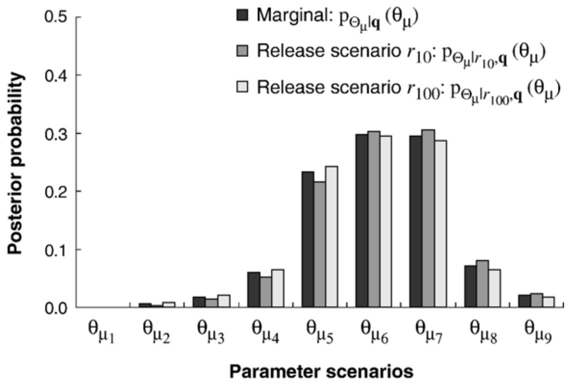

Fig. 4. Posterior probability of Θ_μ, the different scenarios for the model friction parameter μ.

prior probability distributions of the three parameters are assumed to be statistically independent. When parameter values for Θ_μ are available from other sites in the considered region (such as used in Barbolini et al., 2003) those may be reflected in the prior probability model.

The error term in Eq. (5), δ, is assumed to be Normal distributed with mean value $m_\delta=0$ m and standard deviation $s_\delta=150$ m. The influence of this variable is investigated and discussed later.

4.1. Results

In Fig. 4, the posterior marginal probability of the parameter scenario Θ_μ is shown. As described earlier, the posterior probability of Θ_μ is dependent on the release scenario R. To illustrate this dependency, also the posterior probability conditional on r_{10} and r_{100} is included in Fig. 4. It is found that higher values of the friction parameter are more likely for smaller release scenarios, respectively lower snow volumes, which is in accordance with general experience, e.g., Ancey (2005).

Fig. 5 presents the posterior probability of R. Note that the posterior probability of R is similar to the prior

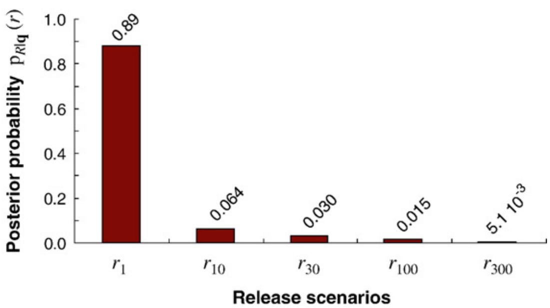

Fig. 5. Posterior probability of R, the different release scenarios.

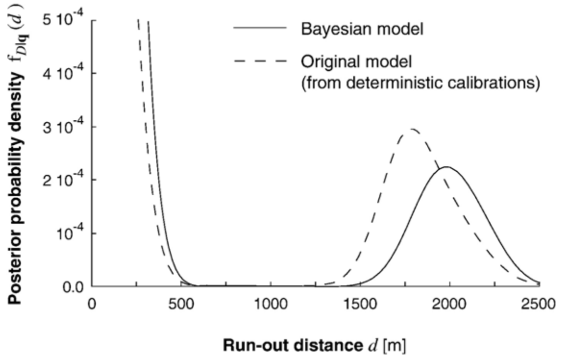

Fig. 6. Pdf of the run-out distance D.

probability. This is explained by the fact that, for the considered area, the release scenario has a less pronounced influence on the run-out distance than the parameter scenario Θ_μ, also see Table 2.

In Fig. 6, the posterior pdf of the run-out distance D is compared to the pdf as calculated with the original model (for which an identical error δ is assumed). The original model is the one traditionally used for the avalanche hazard assessment and is based on a deterministic calibration of the model parameters. It predicts a larger amount of shorter avalanches, whereas the presented probabilistic model predicts fewer but larger avalanches. Note that the model is only conceptual below the threshold $o(t) \leq o_T = 1300$ m and does not reflect the true pdf of avalanches in this range.

The posterior joint probability of Θ_μ and R can be applied in combination with the two-dimensional avalanche model. The results can be expressed by the spatial distribution of the annual probability of exceeding a specific pressure P_{cr} or, alternatively, by the spatial distribution of the pressure which is exceeded with a given probability. Such spatial distributions allow the illustration of the results and form the basis for hazard maps and risk assessments. As an example, Fig. 7 presents the annual probability that an avalanche occurs ($P_{cr} = 0$), as evaluated with the posterior model for the investigated avalanche path.

4.2. Influence of the prior probability distributions

As previously stated, Bayesian analysis requires the specification of the prior probabilities for Θ_μ, Θ_R and Φ. These probabilities should reflect the available information on the parameters before knowledge of the avalanche records used in the Bayesian analysis. When no information is available, a non-informative prior probability distribution should be adopted, see Box and

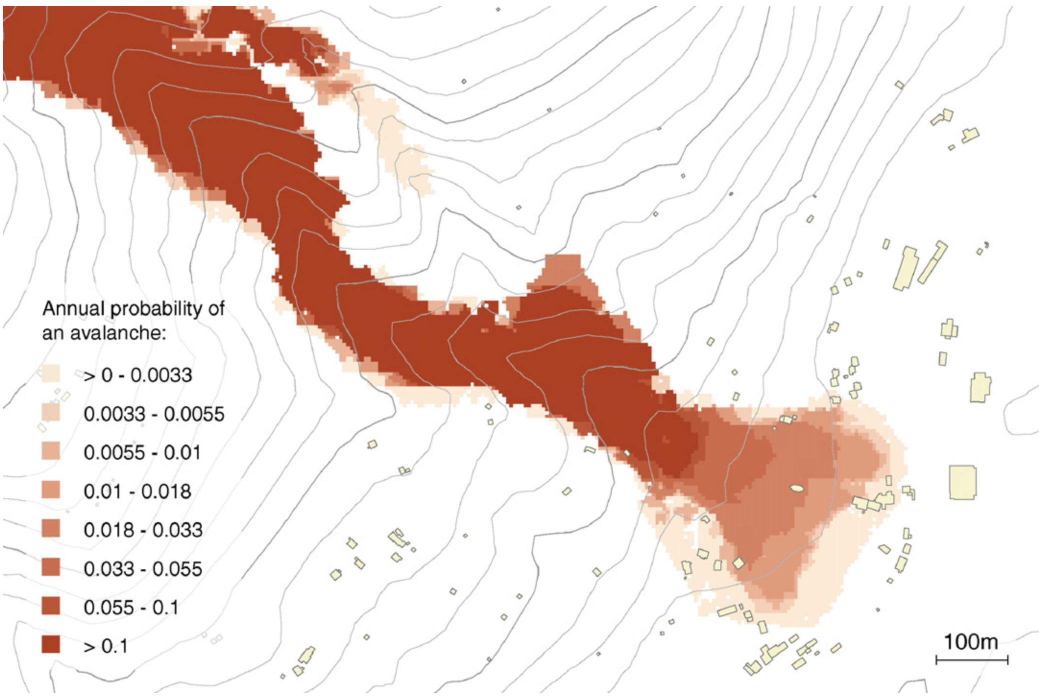

Fig. 7. Annual probability of an avalanche for the considered area (contour lines are shown at 25 m intervals).

Table 4
Alternative prior probability distribution for Θ_μ

$p_{\Theta\mu}(\theta_{\mu 1})$	$p_{\Theta\mu}(\theta_{\mu 2})$	$p_{\Theta\mu}(\theta_{\mu 3})$	$p_{\Theta\mu}(\theta_{\mu 4})$	$p_{\Theta\mu}(\theta_{\mu 5})$	$p_{\Theta\mu}(\theta_{\mu 6})$	$p_{\Theta\mu}(\theta_{\mu 7})$	$p_{\Theta\mu}(\theta_{\mu 8})$	$p_{\Theta\mu}(\theta_{\mu 9})$
0.045	0.065	0.09	0.15	0.3	0.15	0.09	0.065	0.045

Tiao (1992) for details. For the presented avalanche modelling, some information is available from the literature, from observations at other sites and from meteorological data. This information is used to specify the release scenarios R and the model parameter scenarios Θ_μ. In an ideal world, the scenarios specified by the analysts (the original deterministic avalanche model) could be considered as best estimates, having the highest probability of occurrence, with lower probability for the alternative scenarios. Unfortunately (from the viewpoint of the Bayesian analyst), the experts specifying the scenarios also have knowledge on the avalanche records and use those to calibrate the scenarios, in particular the ones describing the friction parameter, Θ_μ. It is difficult to clearly distinguish the different sources of information used by the experts. Therefore, for the determination of the prior probabilities utilized above, some general assumptions have to be made: For the variable Θ_R, the model proposed by the expert is based on snow precipitation statistics, surface, and topography, but not on the observations of avalanches. For this reason, the prior probability presented in Table 3 is chosen, assigning higher probabilities to the scenario specified by the expert ($\Theta_R=3$). For Φ, the annual probability of having an avalanche, we find from the calculations that the posterior probability is dominated by the evidence (i.e., the likelihood term in Bayes' theorem); the result is not sensitive to the choice of the prior probability and the chosen uniform prior probability distribution is therefore appropriate. For Θ_μ, the scenario proposed by the expert is generally obtained by a calibration of the observed avalanches. Therefore, the expert's choice is neglected and a uniform probability distribution is chosen as prior probability for Θ_μ. Note that although uniform probability distributions are commonly applied when non-informative priors are looked for, they are not necessarily non-informative, see Box and Tiao (1992). In the present case they are not non-informative, because discrete scenarios are used and the selection of the scenarios already represents some information.

Because the choice of the prior probabilities is not based on sharp evidence, a sensitivity study is performed to check the influence of the prior probabilities on the final results. To this end, calculations are repeated with different assumptions for the prior probabilities. For Θ_R and Φ, we find that this does not change the posterior probability distributions significantly and therefore the original choice is, appropriate. Changes in the prior probability of Θ_μ do affect the posterior distribution: If the uniform prior probability distribution for Θ_μ is replaced by the distribution summarized in Table 4, the posterior probability of Θ_μ is changing as shown in Fig. 8. The effect on the posterior pdf of the run-out distance D is shown in Fig. 9.

As observed in Fig. 9, the prior probability of Θ_μ does not influence the posterior pdf of the run-out distance significantly, although it does have an impact on the posterior probability of Θ_μ. This at first sight surprising result is caused by the dependency of Θ_μ on R in the posterior model. R also changes accordingly and outweighs the influence of the posterior probability of Θ_μ. As the run-out distance is the relevant quantity in

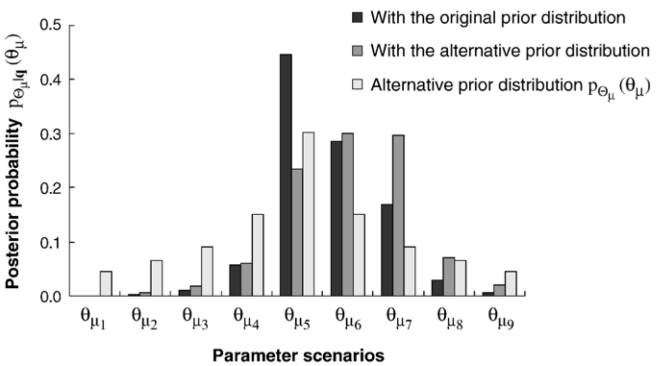

Fig. 8. The effect of a different prior probability distribution on the posterior probability of Θ_μ.

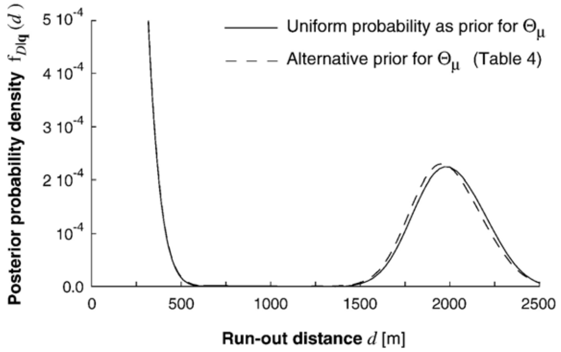

Fig. 9. The effect of the prior probability of Θ_μ on the posterior pdf of the run-out distance D.

the risk assessment, the influence of the prior probability of Θ_μ is not critical for the present case.

In general, the a-priori model has relatively low influence for the investigated case. This is because we have a large set of observations (54 years) including four relevant avalanches. At locations with fewer available observations, the prior probabilities will have a larger influence on the posterior probabilistic model. In those cases it is recommended to use prior probabilities which have been obtained at similar sites in the area or elsewhere.

4.3. The influence of the model error

We introduced an error term in Eq. (2), and respectively Eq. (5). As previously noted, the probability distribution of this error is unknown, and it has been assumed that δ (the additive model and measurement error in the run-out distance) is normal distributed with zero mean and standard deviation $s_\delta = 150$ m. In Fig. 10, the resulting posterior probability of Θ_μ is presented for different values of s_δ. As it is expected, for small values of s_δ, the posterior probability of Θ_μ is concentrated in one single value of Θ_μ. For larger values of s_δ, the prior probability of Θ_μ becomes more important, yet the differences between different values of s_δ are acceptably small. For these reasons, the final choice of $s_\delta = 150$ m is deemed appropriate.

5. Concluding remarks

In this paper, we presented a Bayesian probabilistic framework for avalanche hazard modelling. The framework enables inclusion of observed and expert information in the model and can easily be adapted to include different phenomenological avalanche models. In Grêt-Regamey and Straub (2006) we apply the model to risk assessment in a larger area and find a difference of a factor of two when comparing the resulting annual risk to that calculated using the traditional semi-empirical avalanche hazard model. These differences are relevant, as hazard maps and land-use planning in Switzerland are based on avalanche hazard models; model improvements, therefore, may lead to considerable socio-economical benefits.

Presently available physical and empirical avalanche models are subject to large uncertainties. When using such models for hazard mapping and risk management, additional information is required to support the models. This task is addressed by the proposed framework in

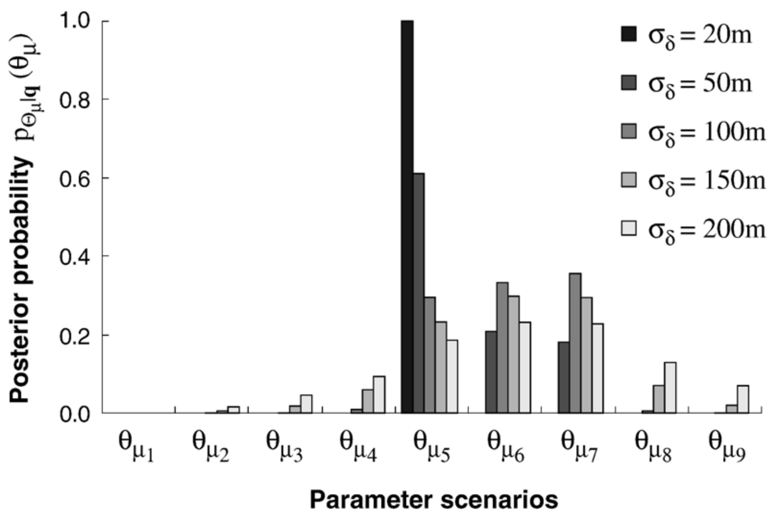

Fig. 10. Influence of different values of σ_δ on the posterior probability of Θ_μ.

analogy to the approach taken by practitioners who (deterministically) calibrate the model parameters to the observations at different sites. The advantage of the Bayesian inference applied here is that it ensures the consistent use of all available information. It explicitly addresses the various uncertainties in the model and quantifies the effect of the observations on these, resulting in an updated probabilistic spatial model of the avalanche hazard. Additionally, the framework facilitates study of the avalanche characteristics in a regional context. As an example, when applying the model to larger areas, thus including larger sets of observations, it will be possible to investigate the dependency of the friction parameter μ, respectively Θ_μ, on the snow release volume, in analogy to previous researchers, see Ancey (2005). Such a dependency is suggested in Fig. 4, but the small number of observations from one particular site used in this paper does not allow any general implications.

The framework has a large potential for further improvements. When the model is applied on a regional scale, the dependency between the different random variables at the different sites must be considered and the model may then be adopted accordingly. As an example, the parameter Φ will be independent from one site to the next, whereas the parameter R (the release scenarios) will be highly dependent because of similar snow precipitation conditions. In this context, it will be of particular relevance to further investigate the dependency structure of the value of the empirical friction parameters at different sites. If no joint regional model is intended, then the information obtained from similar sites may be used to determine the prior probability distributions at the investigated site. This is of particular importance at sites where little information on past events is available.

Furthermore, the presented model can be enhanced by explicitly considering additional input parameters of the dynamic avalanche model as random variables, in analogy to the friction parameter scenarios Θ_μ. If additional random variables are considered, it may also become relevant to consider not only the run-out distance of the avalanches but their complete spatial characteristics. For the presented version of the model, such a spatial updating is not relevant, as the different values of Θ_μ mainly influence the run-out distance (Fig. 1). However, with additional random variables the model may account for the spatial properties in a more flexible way and the Bayesian analysis may then be carried out in two-dimensions. Because the formulation of the likelihood term, Eq. (4), requires the explicit modelling of the model uncertainty, such a spatial analysis requires further investigations on how the spatial model uncertainty (the error term ε in Eq. (2)) can be represented in such a way as to facilitate practical applications. ε is also relevant in the application of the model for risk assessment. In Grêt-Regamey and Straub (2006) we neglect this model error. If systematic errors and biases can be ruled out and if the risk is not concentrated at a single location, we consider this assumption to be reasonable for practical engineering purposes.

Acknowledgments

We thank Urs Gruber from the SLF for providing the AVAL-2D with the data and continuous support. The first author is funded by the Swiss National Science Foundation (SNF) through grant PA002-111428. The second author acknowledges support by the Marie Heim-Vögtlin Fellowship 3234-69265 from the SNF.

References

Ancey, C., 2005. Monte Carlo calibration of avalanches described as Coulomb fluid flows. Philosophical Transactions of the Royal Society, A 363, 1529–1550.

Ancey, C., Gervasoni, C., Meunier, M., 2004. Computing extreme avalanches. Cold Regions Science and Technology 39, 161–180.

Baker, J.W., Cornell, C.A., 2003. Uncertainty Specification and Propagation for Loss Estimation Using FOSM Methods. PEER Report 2003/07, Pacific Earthquake Engineering Research Center.

Barbolini, M., Cappabianca, F., Savi, F., 2003. A new method for the estimation of avalanche distance exceeded probabilities. Surveys in Geophysics 24, 587–601.

Barbolini, M., Gruber, U., Keylock, C.J., Naaim, M., Savi, F., 2000. Application of statistical and hydraulic-continuum dense-snow avalanche models to five real European sites. Cold Regions Science and Technology 31, 133–149.

Barbolini, M., Natale, L., Savi, F., 2002. Effects of release conditions uncertainty on avalanche hazard mapping. Natural Hazards 25, 225–244.

Bartelt, P., Salm, B., Gruber, U., 1999. Calculating dense-snow avalanche runout using a Voellmy-fluid model with active/passive longitudinal straining. Journal of Glaciology 45 (150), 242–254.

Benjamin, J.R., Cornell, C.A., 1970. Probability, Statistics and Decision for Civil Engineers. McGraw-Hill, New York.

Box, G.E.P., Tiao, G.C., 1992. Bayesian Inference in Statistical Analysis. Wiley, New York.

DeGroot, M.H., 1970. Optimal Statistical Decisions. McGraw-Hill, New York.

Faber, M.H., 2005. Uncertainty modelling and probabilities in engineering decision analysis. Journal of Offshore Mechanics and Arctic Engineering, Trans. ASME 127 (3), 243–248.

Grêt-Regamey, A., Straub, D., 2006. Accommodating uncertainties in avalanche risk assessment by linking Bayesian networks to a GIS. Submitted to Natural Hazards and Earth System Sciences.

Gruber, U., 1999. Der Einsatz numerischer Simulationsmethoden in der Lawinengefahrenkartierung. Ph.D. Dissertation, Universtiy of Zurich, Switzerland.

Gruber, U., 2001. Using GIS for avalanche hazard mapping in Switzerland. Proc., 2001 ESRI International User Conference, San Diego. http://gis.esri.com/library/userconf/proc01/professional/papers/pap964/p964.htm.

Hall, J.W., 2003. Handling uncertainty in the hydroinformatic process. Journal of Hydroinformatics 5 (4), 215–231.

Harbitz, C., Harbitz, A., Nadim, F., 2001. On probability analysis in snow avalanche hazard zoning. Annals of Glaciology 32, 290–298.

Jensen, F.V., 2001. Bayesian Networks and Decision Graphs. Springer, New York.

Maggioni, M., Gruber, U., 2003. The influence of topographic parameters on avalanche release dimension and frequency. Cold Regions Science and Technology 37 (3), 407–419.

Pearl, J., 1988. Probabilistic Reasoning in Intelligent Systems. Morgan Kaufmann Publishers, San Mateo, California.

PLANAT, 2004. Sicherheit vor Naturgefahren-Vision und Strategie. Report 7/2004, National Platform for Natural Hazards (PLANAT), Switzerland. www.planat.ch. Available online.

Rackwitz, R., 2000. Optimization — the basis for code-making and reliability verification. Structural Safety 22, 27–60.

Schubert, M., Straub, D., Faber, M.H., 2005. Reliability of rock fall protection galleries — a case study with a special focus on the uncertainty modelling. In: Augusti, et al. (Ed.), Safety and Reliability of Engineering Systems and Structures (Proc. ICOSSAR 05, Rome). Millpress, pp. 1333–1340.

Straub, D., 2005. Natural hazards risk assessment using Bayesian networks. In: Augusti, et al. (Ed.), Safety and Reliability of Engineering Systems and Structures (Proc. ICOSSAR 05, Rome). Millpress, pp. 2535–2542.

Wen, Y.K., Ellingwood, B.R., Veneziano, D., Bracci, J., 2003. Uncertainty modeling in earthquake engineering. Mid-America Earthquake (MAE) Center Project FD-2 Report.